

# The optimal allocating pumping rate of a multi-well system for a brackish water desalination plant

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## Abstract

I applied perspectives on operations research (OR) to propose a non-linear mathematical model, termed the APWs (allocation of pumping wells) model, to address problems associated with allocating pumping rates of a multi-well system for a brackish water reverse osmosis (BWRO) plant. In some arid and inshore regions, feed water mostly preferred to exploit groundwater, owing to its easier desalted characteristic than seawater. However, the operations of most multi-well systems are always arbitrary, and we could not guarantee the strategies of operation might be optimal; additionally, the pumping quantity and quality variation of each well would all induce the unstable quality of feed water, and then lead to the problem of scaling. Though adding acid could treat the problem of scaling, the acidification, on the other hand, means the increasing cost and damage risk of a reverse osmosis (RO) system. Therefore, the overriding objective of this study is to decide the optimal allocating pumping rate among wells for a RO system on the premise of no acid added and under finite groundwater resources, to ensure feed water has no scaling potential and achieve marginal usage of groundwater resources.

The APWs model tries to integrate engineering and management aspects to replace subjective operation of a multi-well system. Here, I apply scaling intensity (SI) proposed by literature that considered calcium carbonate ( $\text{CaCO}_3$ ), calcium sulfate ( $\text{CaSO}_4$ ) and silica ( $\text{SiO}_2$ ) as mainly potential scaling compounds in feed water to assess the scaling potential of a RO system. Besides, I present a solving algorithm for achieving the maximal SI with no scaling occurred. Moreover, I apply the proposed model to a case study in Taiwan by considering four-well system, to obtain optimal allocation with retaining the cleanest residual storage of whole groundwater resources, to avoid the waste of water resources. The results shown that through the assistance of OR technique, it could provide helpful information indeed in RO feed water management. In a word, the APWs model proposed in this paper not only could provide referenced guidelines for all RO operators all over the world, but also allow engineers to gain new insights into their field by integrating different disciplines.

*Keywords:* Scaling; Non-linear programming; Water resources management; Operations research; BWRO

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## 1. Introduction

With the increased water demand from population and economic growth, change in climate and environmental needs, many studies have shown that freshwater shortages are increasing rapidly; in particular in Middle East countries and Mediterranean area [1,2]. The scarcity of conventional freshwater supplies has been a serious threat and a major challenge to sustainable growth and development in the region [2,3]. Fortunately, the mining of non-conventional seawater and brackish water resources could provide an opportunity to water supply sustainability. In fact, desalination plants are being widely used in the Gulf countries as a main source of providing freshwater to overcome the water shortage [4].

However, one of the mainly technique problems associated with desalination reverse osmosis (RO) plant is inorganic compound scaling, and the total dissolve solids (TDS) are always applied to evaluate the water quality. Actually, feed water mostly preferred to exploit brackish water, such as brackish subsurface water (groundwater), owing to its easier desalted characteristic (The TDS concentration in brackish water usually in the range of order  $O(10^2)\sim O(10^3)$  (mg/L) [5]; additionally, the TDS in seawater could always extend the degree of  $O(10^4)$ , some semi-enclosed nature with minimal water exchange with the ocean even reaches as high as  $4.32 \times 10^4$  mg/L [6].) and lower desalination cost (Like Jaber and Ahmed [7] argued that RO has become increasingly attractive for brackish water desalination due to its low-energy requirements. In China, a unit cost of  $\$0.6/\text{m}^3$  for desalting brackish water and  $\$1.0/\text{m}^3$  for seawater are suggested to be appropriate for the potential application [8].) than seawater. Consequently, desalination of brackish groundwater has become a competitive potential water resource to meet potable water demand [2]. Many arid countries, such as Jordan [6], Kuwait [9], Egypt [10] and Iran [2], have exploited the brackish water for many years. However, owing to its

ease-exploitation advantage, over abstraction of groundwater have caused the problem of salinized aquifer seriously (In some regions of the Mediterranean, 58% of coastal aquifers suffer from saline ingress because of over abstraction of groundwater. This has resulted in 25% of the irrigated agricultural land being salinized in some areas [1]. Actually, this is a global problem, Durham et al. [1] further mentioned that 10% of global water usage being sourced from over abstracted groundwater. Twenty percent of global irrigated agricultural areas have been salinized.); therefore the quality of water resources has dramatically deteriorated and consequently it would bring about scaling easily. One of the useful approach to increase the available quantity of feed water is to co-operate with low-salinity groundwater into raw water system, to lower the salinity of mixed brackish water (In fact, we prefer to desalt the lower salinity brackish water. For example, in Kuwait, about 7% portable water was desalinated from the “low-”salinity brackish well [11].), and reduce the membrane scaling potential accordingly. The sketch diagram was shown in Fig. 1; the higher salinity wells exploit coastal aquifers and the relative freshwater is extracted from confined aquifers.

However, because of weak management institutions and regulations, the allocations of exploited raw water resources are always in an uncontrolled and unplanned manner; therefore, they erode the sustainability of water resources. Whether the allocation of the sources of feed water is inadvertent or not, there is a need for guidelines to make the concerned communities better prepared to cope with the increasing water stresses as the storage is depleted. For this reason, I applied perspectives on operations research (OR) to propose a non-linear optimization model, termed the allocation of pumping wells (APWs) model, to address problems associated with allocating pumping rates of a multi-well system for a brackish water reverse osmosis (BWRO) plant.

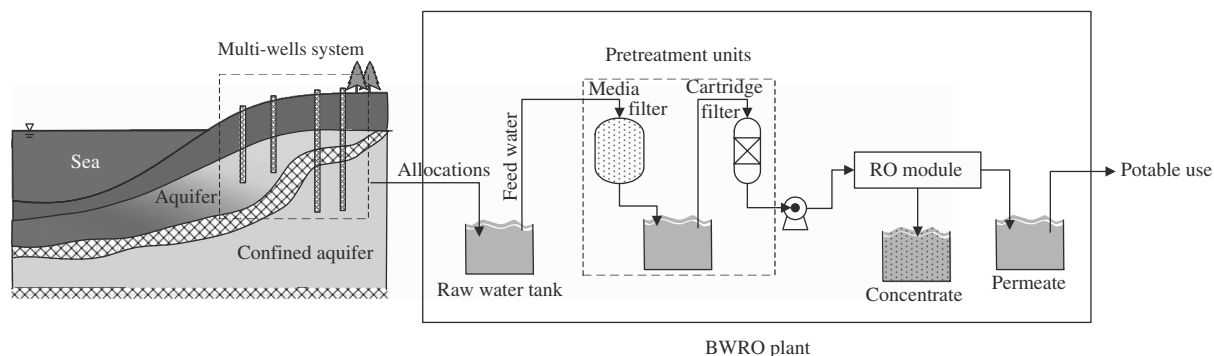


Fig. 1. Diagram of a conjunctive use of higher salinity and relative fresh groundwater for a BWRO plant.

Actually, the APWs model provides a proactive strategy; it tries to integrate engineering and management aspects to replace subjective operation of a multi-well system. Conventional strategies always focus on the technique improvement and lack of adequate water management action, such end-pipe reactive strategies are usually not good for the sustainability of water resources. On the contrary, the proactive strategy that managing the feed water in the pipe-front could avoid the unnecessary waste of precious raw water resources. Additionally, I apply the concept of scaling intensity (SI) proposed by Butt et al. [12], and an easily programmed code proposed by Al-Shammiri et al. [13] for estimating the scaling potential, and in which it considered calcium carbonate ( $\text{CaCO}_3$ ), calcium sulfate ( $\text{CaSO}_4$ ) and silica ( $\text{SiO}_2$ ) as mainly potential scaling compounds in feed water on the premise of no anti-scalant added under finite water resources. Consequently, it could ensure feed water has no scaling potential and achieving marginal usage of raw water system. In short, the APWs model proposed in this paper not only could provide referenced guidelines for all RO operators all over the world, but also allow engineers to gain new insights into their field by integrating different disciplines.

## 2. Model construction and solving algorithms

### 2.1. The constraints

The constraints define a feasible domain in the decision space. In this study, I firstly consider calcium carbonate ( $\text{CaCO}_3$ ), calcium sulfate ( $\text{CaSO}_4$ ) and silica ( $\text{SiO}_2$ ) as the main salts that might precipitate on RO membranes, and the model subjects to no scaling formed on the premise of no anti-scalants added. Besides, I consider the pumping rates capacity and daily water demand to meet the practical applications. Moreover, a simple and easily programmed code proposed by Al-Shammiri et al. [13] was applied to estimate the scaling potential for different scaling species.

#### 2.1.1. Calcium carbonate scaling

Calcium carbonate scaling potential depends on calcium ion concentration, alkalinity of brine stream, K factor, pH of brine stream and pH of saturation [13]. And the Langelier stability index (LSI) was applied to evaluate the scaling potential [12]; if LSI value is negative, this indicates that  $\text{CaCO}_3$  tends to dissolve, but if this value is positive, then the calcium carbonate scaling will form [12]. Several empirical formulations could be described as follows.

$$\text{LSI} = \text{pH}_b - (\text{pH}_{\text{sat}})_b \quad (1)$$

where  $\text{pH}_b$  could be calculated from Eqs. (2) and (3).

$$\text{pH}_b = 0.8426\text{pH}_f + 1.1207 \quad (2)$$

$$\text{pH}_f = -\log \left[ \frac{\sum 10^{-(\text{pH})_j} \cdot Q_j}{\sum Q_j} \right] \quad (3)$$

and  $(\text{pH}_{\text{sat}})_b$  could be calculated from Eqs. (4–7)

$$(\text{pH}_{\text{sat}})_b = p(\text{Ca})_b + p(\text{Alk})_b + K_{\text{factor}} \quad (4)$$

$$p(\text{Ca})_b = -\log(M_{\text{Ca}})_b \quad (5)$$

$$p(\text{Alk})_b = -\log(M_{\text{HCO}_3})_b \quad (6)$$

$$K_{\text{factor}} = -0.7083U_b^2 + 1.8798U_b + 2.1727 \quad (7)$$

where in Eq. (5),  $(M_{\text{Ca}})_b = (M_{\text{Ca}})_f \times \text{CF}$ , and  $(M_{\text{Ca}})_f$  was the mixed concentration of Ca in feed water from various sources. It could be calculated as Eq. (8).

$$(M_{\text{Ca}})_f = \frac{\sum (M_{\text{Ca}})_j Q_j}{\sum Q_j} \quad (8)$$

In Eq. (6), the calculation of bicarbonate ion in the brine stream differed from those of the other ions, depending on the salt passage (SP) of  $\text{HCO}_3^-$  and the type of the permeator used.

Eq. (9) could obtain the concentration of bicarbonate in the brine stream.

$$(M_{\text{HCO}_3})_b = (M_{\text{HCO}_3})_f \times \text{CF} \times (1 - Y \cdot \text{SP}_{\text{HCO}_3}) \quad (9)$$

where the salt passage of  $\text{HCO}_3^-$ ,  $\text{SP}_{\text{HCO}_3}$ , could be determined with Eq. (10).

$$\text{SP}_{\text{HCO}_3} = 2 \cdot 10^9 \text{pH}_f^{-9.80601} \quad (10)$$

and  $(M_{\text{HCO}_3})_f$  could be calculated as Eq. (11)

$$(M_{\text{HCO}_3})_f = \frac{\sum (M_{\text{HCO}_3})_j Q_j}{\sum Q_j} \quad (11)$$

In Eq. (7), the ion strength ( $U_b$ ) of the brine stream could be calculated as Eq. (12); in addition, the Eqs. (13) and (14) were used to convert the computational dimension.

$$U_b = \text{CF} \times U_f = 0.5\text{CF} \cdot \sum (M_{\text{li}})_f Z_i^2 \quad (12)$$

$$(M_{\text{li}})_f = \frac{(M_i)_f \cdot 10^6}{10^6 - \text{TDS}_f} \quad (13)$$

$$(M_i)_f = \frac{(C_i)_f}{1000 \cdot W_i} \quad (14)$$

$$\text{and } \text{TDS}_f = \frac{\sum \text{TDS}_j Q_j}{\sum Q_j}$$

To be more specific, Eq. (12) could be rewritten as

$$U_b = 0.5\text{CF} \cdot \frac{\left( (M_{\text{HCO}_3^-} + M_{\text{Na}^+} + M_{\text{K}^+} + M_{\text{NH}_4^+} + M_{\text{F}^-} + M_{\text{Cl}^-} + M_{\text{NO}_3^-})_f \times 1^2 + (M_{\text{Ca}^{2+}} + M_{\text{Sr}^{2+}} + M_{\text{Ba}^{2+}} + M_{\text{Fe}^{2+}} + M_{\text{Mg}^{2+}} + M_{\text{Mn}^{2+}} + M_{\text{Cu}^{2+}} + M_{\text{Zn}^{2+}} + M_{\text{CO}_3^{2-}} + M_{\text{SO}_4^{2-}})_f \times 2^2 + (M_{\text{Al}^{3+}} + M_{\text{PO}_4^{3-}})_f \times 3^2 \right)}{10^6 - \text{TDS}_f} \times 10^6$$

We should note that the concentration of ions ( $C_i$ ) used in the calculation was expressed in mg/L; only bicarbonate and carbonate ions have been converted to mg/L as  $\text{CaCO}_3$ .

Consequently, to avoid the calcium carbonate scaling, the LSI value of brine water should be negative, namely  $\text{pH}_b - (\text{pH}_{\text{sat}})_b < 0$ ; then we combined the Eqs. (2–7) and rearrange them, the constraint of inhibit calcium carbonate scaling could be obtained as

$$\log(M_{\text{Ca}})_b + \log(M_{\text{HCO}_3})_b + 0.7083U_b^2 - 1.8798U_b - 2.1727 < -0.8426\text{pH}_f - 1.1207 \quad (15)$$

where  $\log(M_{\text{Ca}})_b + \log(M_{\text{HCO}_3})_b + 0.7083U_b^2 - 1.8798U_b - 2.1727$  (i.e.,  $-(\text{pH}_{\text{sat}})_b$ ) was called the negative critical value of  $\text{CaCO}_3$  scaling potential, and  $-0.8426\text{pH}_f - 1.1207$  (i.e.,  $-\text{pH}_b$ ) is the negative current scaling potential value of  $\text{CaCO}_3$ ; Additionally, we define  $\frac{\text{pH}_b}{(\text{pH}_{\text{sat}})_b}$  as the SI index for calcium carbonate scaling (abbreviated as  $\text{SI}_{\text{for CaCO}_3}$ ); if the value is less than 1, then no scaling would occur.

### 2.1.2. Calcium sulfate scaling

The ion product (IP) of calcium sulfate scaling is calculated with Eq. (16), where  $(M_{\text{Ca}})_b$  and  $(M_{\text{SO}_4})_b$  could be obtained by Eq. (8) multiplies CF and Eq. (17), respectively. Then with comparing to the compound solubility product (Ksp), Eq. (18), the constraint of inhibit calcium sulfate scaling could be easily found [Eq. (19)], i.e.,  $(\text{IP}_{\text{CaSO}_4})_b < (\text{Ksp}_{\text{CaSO}_4})_b$ . Similarity to calcium carbonate, we define  $\frac{(\text{IP}_{\text{CaSO}_4})_b}{(\text{Ksp}_{\text{CaSO}_4})_b}$  as the SI index for calcium sulfate scaling (i.e.,  $\text{SI}_{\text{for CaSO}_4}$ ); and if the value is less than 1, no scaling would occur.

$$(\text{IP}_{\text{CaSO}_4})_b = (M_{\text{Ca}})_b \cdot (M_{\text{SO}_4})_b \quad (16)$$

$$(M_{\text{SO}_4})_b = \frac{\sum (M_{\text{SO}_4})_j \cdot Q_j}{\sum Q_j} \times \text{CF} \quad (17)$$

$$(\text{Ksp}_{\text{CaSO}_4})_b = 0.0016U_b^{0.6742} \quad (18)$$

$$(M_{\text{Ca}})_b \cdot (M_{\text{SO}_4})_b < 0.0016 \cdot U_b^{0.6742} \quad (19)$$

### 2.1.3. Silica scaling

For silica scaling, if  $(C_{\text{SiO}_2})_b$  is greater than  $(C_{\text{SiO}_2})_{\text{lit}}$ , then there is a chance of scaling; but if the reverse is true then no scaling is expected. Namely, to avoid silica scaling, the Eq. (20) should be satisfied.

$$(C_{\text{SiO}_2})_b < (C_{\text{SiO}_2})_{\text{lit}} \quad (20)$$

Besides, some empirical formulations relative to Eq. (20) could be obtained as follows.

$$(C_{\text{SiO}_2})_{\text{temp}} = 1.9872 \text{ Temp} + 75.37 \quad (21)$$

$$(C_{\text{SiO}_2})_{\text{lit}} = (C_{\text{SiO}_2})_{\text{temp}} \cdot \text{PHCF} \quad (22)$$

where PHCF, represented in Eq. (22), is the pH correction factor for silica scaling potential, and

$$\text{PHCF} = 0.229 \times (\text{pH}_b)^2 - 2.8803 \times \text{pH}_b + 9.6889 \quad (23)$$

To be more specific, the constraint of inhibit silica scaling could be obtained by rearranging Eqs. (20–23).

$$(C_{\text{SiO}_2})_b < (1.9872 \times \text{Temp} + 75.37) \times [0.229 \times (\text{pH}_b)^2 - 2.8803 \times \text{pH}_b + 9.6889] \quad (24)$$

$$(C_{\text{SiO}_2})_b = \frac{\sum (C_{\text{SiO}_2})_j \cdot Q_j}{\sum Q_j} \times \text{CF} \quad (25)$$

Besides, the SI index for silica scaling (i.e.,  $\text{SI}_{\text{for SiO}_2}$ ) was defined as  $\frac{(C_{\text{SiO}_2})_b}{(C_{\text{SiO}_2})_{\text{lit}}}$ , and if the value is less than 1, no scaling would occur.

## 2.2. The optimal APWs model

This model regards all raw water resources as a whole and considers the maximal SI. Therefore, the operating strategy is to retain the cleanest residual storage of whole groundwater resources. With integrating the objective function and the constraints, the model was shown in P1, it reflects that the strategy of operation is to obtain allocation with dirtiest no scaling feed water and to retain the quality of whole groundwater resources as far as possible.

The first three constraints shown in P1 are the thresholds of no scaling formed in the system, they also imply that the allocation of pumping rates should meet the following inequalities, i.e.,  $\text{pH}_b < (\text{pH}_{\text{sat}})_b$ ,  $(\text{IP}_{\text{CaSO}_4})_b < (\text{Ksp}_{\text{CaSO}_4})_b$ , and  $(C_{\text{SiO}_2})_b < (C_{\text{SiO}_2})_{\text{lit}}$  [refers to Eqs. (15), (19) and (24)]; Therefore, I apply the normalization technique to assess the scaling potential, for instance  $\frac{(\text{IP}_{\text{CaSO}_4})_b}{(\text{Ksp}_{\text{CaSO}_4})_b}$  regards as the inhibition potential

intensity for calcium sulfate scaling, and if the value is less than 1, no calcium sulfate scaling would occur. Consequently, the scaling intensity index, SI, could be shown as:

$\frac{\text{pH}_b}{(\text{pH}_{\text{sat}})_b} + \frac{(C_{\text{SiO}_2})_b}{(C_{\text{SiO}_2})_{\text{lit}}} + \frac{(\text{IP}_{\text{CaSO}_4})_b}{(\text{Ksp}_{\text{CaSO}_4})_b}$ , and the theoretical no scaling upper bound is 3. In fact, SI index is dimensionless, and the value reflects the level of dirty degree of feed water. In other words, the lower SI means more precious water resources (from water quality perspective). Additionally, the fourth constraint means the decision pumping rates should meet the pumping capacity for each well, and the final constraint represents that the total pumping rates satisfy daily demand.

Moreover, the more concrete description of the above mentioned objective function could be shown in Fig. 2. The points A and B indicate that these allocations of pumping rates would form scaling certainly, and points C–F would not necessarily occur. In addition, the point D is most close to the upper bound of no scaling (although point C is closer than point D; however it forms scaling), namely the strategy of point D could obtain the dirtiest feed water without scaling occurred, and retain the quality of whole groundwater resources to avoid its wasteful usage. Besides, we should keep in mind that even though the point E and F would not induce scaling;

$$\begin{aligned} \max_Q & \frac{\text{pH}_b}{(\text{pH}_{\text{sat}})_b} + \frac{(C_{\text{SiO}_2})_b}{(C_{\text{SiO}_2})_{\text{lit}}} + \frac{(\text{IP}_{\text{CaSO}_4})_b}{(\text{Ksp}_{\text{CaSO}_4})_b} \\ \text{s.t.} & \\ & \log(M_{\text{Ca}})_b + \log(M_{\text{HCO}_3})_b + 0.7083U_b^2 - 1.8798U_b - 2.1727 < -0.8426\text{pH}_f - 1.1207 \\ \text{APWs model} & (C_{\text{SiO}_2})_b < (1.9872 \times \text{Temp} + 75.37) \cdot \left[ 0.229 \times (\text{pH}_b)^2 - 2.8803 \times \text{pH}_b + 9.6889 \right] \quad (\text{P1}) \\ & (M_{\text{Ca}})_b \cdot (M_{\text{SO}_4})_b < 0.0016 \cdot U_b^{0.6742} \\ & (Q_{\text{min}})_j \leq Q_j \leq (Q_{\text{max}})_j \\ & \sum Q_j = \tilde{Q} \end{aligned}$$

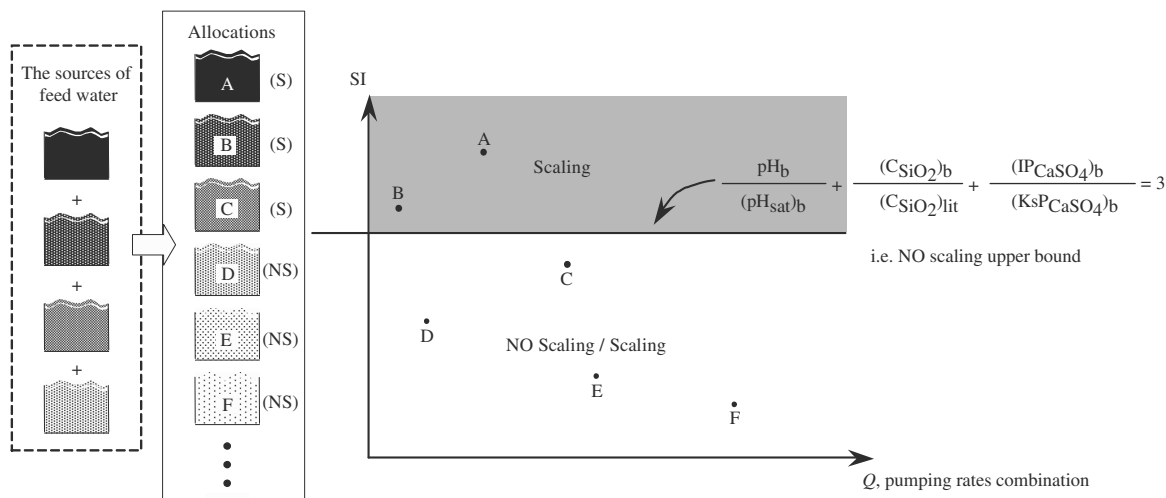


Fig. 2. The sketch diagram that selects SI index as objective for a RO plant. *Note:* Scaling is denoted by (S), and NO scaling is denoted by (NS).

however, selecting them implies that we adopt a wasteful strategy.

### 2.3. Solving algorithms

For solving APWs model, the FORTRAN program was applied to conduct the model and a two-staged solving procedure has been developed. In the first stage, we applied the concept of threshold-scaling with meeting the pumping rates capacity to elect the candidates; and in the second stage, the optimal solution was found by wide-ranging searching method from candidates solutions (feasible domain). In addition, the objective of the model is to retain the quality of whole groundwater resources as far as possible.

## 3. Numerical case study

In this section, I provided a case study of a BWRO plant located in Bai-sha village in Penghu county, Taiwan. The Penghu county is off the western coast of Taiwan in the Taiwan Strait consisting of 64 small islands. The average annual rainfall is only 1000 mm, however, the annual amount of evaporation can reach 1600 mm.

Therefore, the scarcity of conventional freshwater supplies infers a serious threat to sustainable and balanced socio-economic growth and development. To deal with the problem of water shortage, the government has conducted several programs including constructing a groundwater reservoir and several desalination plants. Although groundwater is one of the major water resources for domestic use, the over abstraction of the aquifers has caused the seawater intrusion and have increased salinity of such well water (usually shallow unconfined aquifer). Therefore, the conjunctive use of contaminated aquifers and deep cleaner confined aquifers for a BWRO plant was commonly applied.

The study site, Bai-sha brackish water desalination plant, has two different sources of feed water; one from the Chih-Kan Ground Reservoir, a subsurface reservoir, and the other from deep confined aquifer. Where the Chih-Kan Ground Reservoir is classified as a shallow unconfined aquifer and its TDS is around 1500 mg/L. In contrast, the deep confined aquifer has cleaner raw water, and its TDS is about 500 mg/L. The detailed raw groundwater parameters of each well could be shown in Table 1. The recovery rate ( $Y$ ) is 0.6, the

Table 1

Raw groundwater characteristics from Chih-Kan Ground Reservoir (W001 and W002) and deeper confined aquifer (W003 and W004) treated by the BWRO plant

Parameters	Unit	W001	W002	W003	W004
pH	–	8.3	8.0	8.3	8.1
TDS	mg/L	1610	1530	560	480
$C_{\text{HCO}_3^-}$	mg/L	138	116	82	65
$C_{\text{CO}_3^{2-}}$	mg/L	1.3	1.2	1.4	1.1
$C_{\text{SO}_4^{2-}}$	mg/L	33.6	32	56	81
$C_{\text{PO}_4^{3-}}$	mg/L	0.051	0.05	0.03	0.03
$C_{\text{F}^-}$	mg/L	0.09	0.07	0.09	0.08
$C_{\text{Cl}^-}$	mg/L	500	780	340	320
$C_{\text{Ca}^{2+}}$	mg/L	79.2	84	7.5	10.2
$C_{\text{Mg}^{2+}}$	mg/L	35.2	37.8	8.8	5.6
$C_{\text{Sr}^{2+}}$	mg/L	0.825	0.7	0.8	0.6
$C_{\text{Ba}^{2+}}$	mg/L	0.026	0.028	0.07	0.05
$C_{\text{Fe}^{2+}}$	mg/L	0.058	0.03	0.04	0.03
$C_{\text{Mn}^{2+}}$	mg/L	0.107	0.161	0.06	0.03
$C_{\text{Na}^+}$	mg/L	280	435	280	270
$C_{\text{NH}_4^+}$	mg/L	0.44	0.14	0.4	0.3
$C_{\text{Al}^{3+}}$	mg/L	0.006	0.007	0.07	0.04
$C_{\text{SiO}_2}$	mg/L	24.3	15.7	47	51
Operation temperature	°C	25			
Product recovery	%	60			
Permeate	CMD	1080			

total needed pumping quantity from all wells is 1800 CMD, namely the RO plant produce 1080 m<sup>3</sup> water per day; The minimum/maximum pumping capacity of each well is between 0–850 CMD.

#### 4. Results and discussions

After performing the APWs model with FORTRAN program, the computer simulates 614,125 (i.e., 85<sup>3</sup>) times with the pumping rate increment is 10 CMD, and has total 74 candidates solutions with no scaling occurred. The suggested allocations of pumping rates for each well are shown in Table 2.

From Table 2, there have been several implications for managers. First, the scaling of CaCO<sub>3</sub>

is the dominant inorganic compound in Bai-sha BWRO plant, since the scaling index,  $SI_{\text{for CaCO}_3}$ , is most approach to its non-scaling upper bound (i.e., 1). Similarly, SiO<sub>2</sub> is also sensitive to the system ( $SI_{\text{for SiO}_2} \approx 0.7 \rightarrow 1$ ); however, the scaling of calcium sulfate is not dominant, as a result of  $SI_{\text{for CaSO}_4}$  equals 0.00529. In other words,

$$\frac{(IP_{\text{CaSO}_4})_b}{(K_{\text{sp CaSO}_4})_b} \ll 1; \text{ consequently no calcium sul-}$$

fate scaling formed [refers to Eq. (22)]. Although the theoretical non-scaling upper bound SI is 3, the suggested allocations could only achieve 1.7252; that is because in this BWRO system, only both calcium carbonate and silica are sensitive to form scaling.

Table 2

The allocations of pumping rates that suggested by APWs model and conventional strategies and corresponding indices

$Q_1$	$Q_2$	$Q_3$	$Q_4$	$\frac{\text{pH}_b}{(\text{pH}_{\text{sat}})_b}$	$\frac{(C_{\text{SiO}_2})_b}{(C_{\text{SiO}_2})_{\text{lit}}}$	$\frac{(\text{IP}_{\text{CaSO}_4})_b}{(\text{Ksp}_{\text{CaSO}_4})_b}$	$U_b$	SI	Scaling	Remarks
110	0	840	850	0.99852	0.71820	0.00849	0.0351	1.7252	No	Optimal <sup>a</sup>
60	50	850	840	0.99766	0.71724	0.00846	0.0356	1.7234	No	Candidates <sup>b</sup>
10	120	820	850	0.99968	0.71364	0.00891	0.0365	1.7222	No	Candidates <sup>c</sup>
300	400	500	600	–	–	–	–	>3	Yes	Arbitrary given

1. SI means the scaling intensity, and was defined as  $\frac{\text{pH}_b}{(\text{pH}_{\text{sat}})_b} + \frac{(C_{\text{SiO}_2})_b}{(C_{\text{SiO}_2})_{\text{lit}}} + \frac{(\text{IP}_{\text{CaSO}_4})_b}{(\text{Ksp}_{\text{CaSO}_4})_b}$ ; besides the theoretical no scaling upper bound is 3, i.e., if  $\text{SI} > 3$ , then scaling occurs undoubtedly.

2.  $U_b$  means the ionic strength of brine water, and has relation with calcium carbonate scaling and calcium sulfate scaling.

Second, the exploited unconfined aquifer (Chih-Kan Ground Reservoir) contains unsuitable feed water composition for RO desalination technique since they were characterized by higher concentrations of divalent ions, which increased the requirements of anti-scalant additives [9]. Moreover, through simulations, we find that the cation  $\text{Ca}^{2+}$  and the anion  $\text{HCO}_3^-$  are the most scaling-effective ions in this BWRO plant (ref. to  $\text{SI}_{\text{for CaCO}_3}$ ). Actually, the membrane scaling is not just associated with the TDS but also affected by the concentration of all ions in the feed water.

Third, on the premise of no anti-scalant added, the no scaling combinations suggested by the proposed model shown that the pumping rates of  $Q_1$  is 0–110 CMD,  $Q_2$ : 0–130 CMD,  $Q_3$  and  $Q_4$  are all between 820–850 CMD. It is useful for operator to refer. In fact, past subjective allocations, such as shown in Table 2: arbitrary given (cf. Fig. 2, point A or B), Candidates<sup>a</sup> (cf. Fig. 2, point F) and Candidates<sup>b</sup> (cf. Fig. 2, point E), they not only cause the waste of raw water resources, but increase unnecessary cost, like anti-scalant additives and membrane damage.

The contribution of this study could be concluded as that a concrete mathematical model was constructed to describe a practical BWRO feed water management problem. Just as Hamoda [5]

argued: “a proactive approach has to be developed to ensure the sustainable management of all resources.” Indeed, the proposed model in this paper is an efficient and comprehensive planning tool for managers to decide better strategies and could provide new insights to engineers in their field.

## Nomenclature

### Parameters

$C_i$	concentration of ion (mg/L)
CF	the concentration factor, $\text{CF} = \frac{1}{1 - Y}$ (i.e., the so-called concentration rate)
IP	the ion product
Ksp	solubility product
$M_i$	molarity concentration of ions (moles/L)
$n$	the total number of available wells of a raw water system
PHCP	the pH correction factor for silica scaling potential
$\tilde{Q}$	the daily demand of feed water for a RO plant
$(Q_{\text{min}})_j$	the minimum allowable pumping rate of well $j$
$(Q_{\text{max}})_j$	the maximum allowable pumping rate of well $j$
TDS	total dissolved solids as mg/L
Temp	temperature (°C)

$U$	ionic strength
$W_i$	molecular weight of ions in g/mol
$Y$	the conversion (i.e., the so-called recovery rate)

#### Decision variables

$Q$	$Q$ , is a vector showing the probable combination of pumping rates with all wells, i.e., $Q = (Q_1, Q_2, \dots, Q_j, \dots, Q_n)$ , besides $Q_j$ means the component of $Q$ vector
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#### Subscripts and superscripts

b	brine water
f	feed water
i	ion
$l$	molal concentration as moles/1000 g of water
$j$	wells number, $\forall j = 1, 2, \dots, n$

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