

Design and operating characteristics of pilot scale reverse osmosis plants

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Abstract

Design and operating characteristics are analyzed for two pilot scale reverse osmosis plants. The first plant is installed in Sharjah to desalinate brackish water for a small community. The second plant is installed in Qatar to test the feasibility of desalting high-salinity seawater for irrigation purposes. Analysis is performed using a semi-empirical model, which requires knowledge of membrane salt reject and permeate recovery. Further analysis is made by using a permeability model, which is capable of predicting the required membrane area. Predictions of the two models are in good agreement with the available field data. Both models can provide the plant engineers with a useful tool to assess plant performance and analyze deviations from original design conditions.

Keywords: Reverse osmosis; Modeling; Design; Simulation

1. Introduction

Although the desalination industry in the Gulf area is dominated by the multistage flash desalination process [1], serious attempts are being made to adopt the more efficient reverse osmosis (RO) desalination process. In this regard, several small-scale RO plants are installed to desalinate either brackish water (BWRO) or high-salinity seawater (SWRO). Progress review of the RO process

by Matsuura [2] shows simultaneous increase in salt rejection and normalized flux, where the salt rejection reached values close to 99.5% and a normalized flux of 2 m³/m² day MPa is achieved. Other developments include synthesis of membranes capable of withstanding high operating pressure for long periods of operation and higher resistance to chemical and biological fouling.

This study focuses on modeling and analysis of the performance of two pilot scale RO plants. Review of literature shows several approaches for RO modeling, which includes the irreversible

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thermodynamic model, the frictional model, the solution-diffusion model, the solution-diffusion-imperfection model, the preferential adsorption capillary flow model, the diffusion viscous flow model, and the finely porous model. All these models are special cases of the statistical-mechanical model originally developed by Mason and Lonsdale [3]. Soltanieh and Gill [4] gave a review for several mass transfer models of the RO system. Analyses of the spiral wound membranes have been made by Dickson et al. [5], Taniguchi [6], and Rautenbach and Dahm [7]. Similarly, Gill and Bansal [8], Hermans [9], Kabadi et al. [10], Ohya et al. [11], and Sekino [12] used these models to study the hollow fine fiber system.

The models used in this study include a semi-empirical model, which is based on knowledge of membrane salt rejection and membrane permeate recovery ratio as well as performing total mass and salt balances on the membrane module. In addition, the permeability model is used to predict the required permeation area. The next sections include review of RO modeling parameters, model assumptions, model equations, modeling of the two RO plants, and comparison of model predictions against field data.

2. RO modeling parameters

Characteristics of modern RO plants include the following:

- The total production capacity of RO plants may vary from few cubic meters per day up to 100,000 m³/day [1].
- The membrane salt rejection ratio, which is the ratio for the salinity difference of feed and permeate product to the salinity of the feed stream, may vary over a range of 95–99.5%. Use of membranes with high salt rejection is necessary for seawater. This is to maintain the product salinity within limits of potable water characteristics, i.e., below 1000 ppm.
- The module permeate recovery ratio, which defines the flow rate ratio of the permeate

product and the feed stream, may vary from 5% to 10% for each membrane module and from 30% to 90% for the entire plant. In seawater desalination, recovery ratios are limited to values below 50–60%, while for low salinity brackish water recovery may reach 90%.

- The feed pressure depends on the feed salinity. The feed pressure must account for the osmotic pressure generated by the salinity on the feed side as well as module friction losses and membrane resistance. The feed pressure should be high enough to generate high permeate flow rates. For seawater desalination, feed pressures up to 7500 kPa are common, while in brackish water desalination the feed pressure might be as low as 500 kPa.

3. Model assumptions

The assumptions used to develop the RO models include the following:

- Steady state operation.
- Isothermal operation. Therefore, the temperatures of the feed, brine, and permeate are equal.
- The membrane selectivity is constant and is equal for various types of salts.
- The semi-empirical model assumes constant permeate recovery and salt rejection for each membrane module.
- The permeability model assumes constant permeability coefficient for water and salt permeation across the membrane.
- Complete mixing conditions are assumed in the feed and permeate compartments in each membrane module.
- Permeate pressure is constant and equal to 101 kPa.

3.1. Semi-empirical model

The semi-empirical model is based on measurements of the membrane salt rejection and product recovery, which are given by

$$Q_p = RR Q_f \tag{1}$$

$$X_p = X_f(1 - SR) \tag{2}$$

Other model equations included mass and salt balances on the membrane module.

$$Q_f = Q_b + Q_p \tag{3}$$

$$X_f Q_f = X_b Q_b + X_p Q_p \tag{4}$$

The following relations give the pressure drop across the membrane and the osmotic pressure difference between the feed and permeate side. It should be noted that the pressure difference across the membrane should be greater than the osmotic pressure difference and other forms of losses caused by friction and the membrane resistance.

$$\Delta p_m = 0.5(p_f + p_b) - p_p \tag{5}$$

$$\Delta \pi_m = RT \frac{\rho}{MW} (\bar{X} - X_p) \tag{6}$$

Where the average salinity on the feed side is given by the following relation:

$$\bar{X}_b = \frac{X_b Q_b + X_p Q_p}{Q_b + Q_p} \tag{7}$$

Eqs. (1–7) are solved sequentially to determine the permeate flow rate and salinity, the brine flow rate and salinity, the pressure drop across the membrane, and the osmotic pressure difference across the membrane. The model equations can be applied to a set of modules in series, where the brine or the permeate stream from one module is the feed stream to the next module.

3.2. Permeability model

The permeability model is a special case of the statistical-mechanical model, originally developed by Mason and Lonsdale [3]. The model includes total mass balance, salt balance, water flux across the membrane, and salt flux across the membrane. The permeator mass and salt balances are given by Eqs. (3) and (4). The water flux across the membrane is given by

$$Q_p = (\Delta p_m - \Delta \pi_m) P_w A \tag{8}$$

where the membrane pressure drop (Δp_m) and the osmotic pressure difference across the membrane ($\Delta \pi_m$) are given by Eqs. (5) and (6).

The salt flux across the membrane is given by the following relation:

$$X_p Q_p \rho_p 10^{-6} = (\bar{X} - X_p) P_s A \tag{9}$$

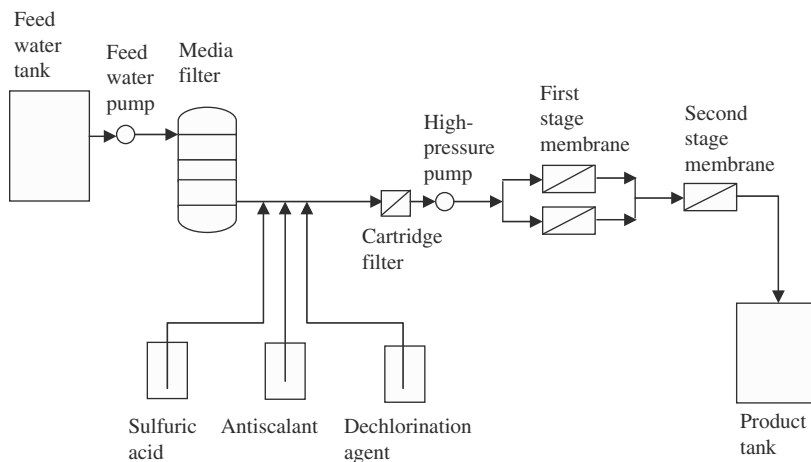


Fig. 1. Schematic of Sharjah potable water RO plant.

In Eq. (10), the term \bar{X} is the average salinity on the feed side and is given by Eq. (7).

Eqs. (3), (4), (8), and (9) are solved simultaneously to calculate the membrane area, the permeate salinity, and the brine salinity and flow rate. Solution of this set of equations requires definition of the salt and water permeability, feed pressure, feed flow rate and salinity, the system temperature, and the recovery ratio or the permeate flow rate.

3.3. Performance analysis of the Sharjah BWRO plant

The Sharjah BWRO plant has a small production capacity of 237.5 m³/day. The plant layout is shown in Fig. 1. The feed salinity is 3200 ppm and the permeate salinity is 353 ppm. The system contains two stages to maximize the system recovery.

Table 1 shows comparison of plant data and the predictions of the semi-empirical model. It should

Table 1
Comparison of actual field data and predictions of the semi-empirical model for the Sharjah plant

Parameter	Actual	Model
Total permeate flow rate (m ³ /day)	237.5	237.5
Feed salinity (ppm)	3500	3500
Permeate flow rate per module in first stage (m ³ /day)	NA	1.031248
Salt rejection per module in first stage	0.9163	0.94
Permeate recovery per module in first stage	0.12	0.097594
Feed pressure to first stage (kPa)	2200	2200
Pressure drop per module in first stage (kPa)	NA	24
Permeate flow rate per module in second stage (m ³ /day)	NA	1.711834
Salt rejection per module in second stage	0.9163	0.94
Permeate recovery per module in second stage	0.12	0.12
Feed pressure to second stage (kPa)	1800	1800
Pressure drop per module in second stage (kPa)	NA	24
Permeate pressure (kPa)	101	101
Temperature (°C)	25	25
Average molecular weight	NA	33
<i>Design results</i>		
Ratio of number of vessels in first to second stage	2.5	2.5
Number of vessels in first stage	30	30
Number of vessels in second stage	12	12
Total permeate flow rate from first pass per pressure vessel (m ³ /day)	4.86	4.86
Total permeate flow rate from second pass per pressure vessel (m ³ /day)	7.64	7.64
Number of modules in first stage	180	180
Number of modules in second stage	72	72
Average permeate salinity from first stage (ppm)	263.45	263.45
Average permeate salinity from second stage (ppm)	495.66	495.66
Product salinity (ppm)	353.10	353.10
Total system recovery (%)	75	75
Total feed flow rate (m ³ /day)	317	317

Table 2a
Predicted profiles by the semi-empirical model for the first stage

Module number	1	2	3	4	5	6
Q_f (m ³ /day)	10.57	9.54	8.60	7.77	7.01	6.32
Q_b (m ³ /day)	9.54	8.60	7.77	7.01	6.32	5.71
Q_p (m ³ /day)	1.03	0.93	0.84	0.76	0.68	0.62
X_p (ppm)	210	231.3486	254.8675	280.7773	309.3212	340.7668
X_f (ppm)	3500.00	3855.81	4247.79	4679.62	5155.35	5679.45
X_b (ppm)	3855.81	4247.79	4679.62	5155.35	5679.45	6256.82
\bar{X}_b (ppm)	3668.78	4041.75	4452.63	4905.28	5403.96	5953.32
p_f (kPa)	2200	2176	2152	2128	2104	2080
p_b (kPa)	2176	2152	2128	2104	2080	2056
$\Delta\pi_m$ (kPa)	259.82	286.23	315.33	347.39	382.70	421.61
Δp_m (kPa)	1827.18	1776.77	1723.67	1667.61	1608.30	1545.39

Table 2b
Predicted profiles by the semi-empirical model for the second stage

Module number	1	2	3	4	5	6
Q_f (m ³ /day)	14.27	12.55	11.05	9.72	8.55	7.53
Q_b (m ³ /day)	12.55	11.05	9.72	8.55	7.53	6.62
Q_p (m ³ /day)	1.71	1.51	1.33	1.17	1.03	0.90
X_p (ppm)	375.41	423.53	477.82	539.07	608.16	686.12
X_f (ppm)	6256.82	7058.83	7963.64	8984.44	10,136.08	11,435.34
X_b (ppm)	7058.83	7963.64	8984.44	10,136.08	11,435.34	12,901.14
\bar{X}_b (ppm)	6632.23	7482.36	8441.46	9523.50	10,744.24	12,121.46
p_f (kPa)	1800	1776	1752	1728	1704	1680
p_b (kPa)	1776	1752	1728	1704	1680	1656
$\Delta\pi_m$ (kPa)	470.00	530.25	598.22	674.90	761.41	859.01
Δp_m (kPa)	1217.00	1132.75	1040.78	940.10	829.59	707.99

Table 3
Simulation parameters for the permeability model of the Sharjah plant

Simulation parameter (units)	Value
Membrane area (m ²)	0.01
Salt permeability (kg/s ppm m ²)	6×10^{-8}
Water permeability (m ³ /s kPa m ²)	7×10^{-7}
Permeate pressure (kPa)	101
System temperature (°C)	25
Average molecular weight of salt (kg/kmole)	33
Pressure drop per module (kPa)	24

be made clear that to achieve good agreement between plant data and model predictions the module recovery and salt rejection in the first stage are adjusted. Other design values, which are not available (NA) from the plant data, included the average molecular weight for the dissolved salts in the feed, permeate flow rate per module, and pressure drop per module in the first and second stages. As is shown in the design results, excellent agreement is obtained between the field data and the model predictions. The calculated system profiles, which include flow rates, salinity, pressure, and osmotic pressure across the modules

Table 4a
Predicted profiles by the permeability model for the first stage in Sharjah plant

Module number	1	2	3	4	5	6
Q_f (m ³ /s)	1.22E-04	1.10E-04	9.72E-05	8.52E-05	7.37E-05	6.28E-05
Q_b (m ³ /s)	1.10E-04	9.72E-05	8.52E-05	7.37E-05	6.28E-05	5.26E-05
Q_p (m ³ /s)	1.28E-05	1.24E-05	1.20E-05	1.15E-05	1.09E-05	1.02E-05
X_f (ppm)	3500	3888.4	4359.6	4940.2	5667.6	6595.5
X_p (ppm)	165.4	190.0	221.3	262.4	318.0	396.1
X_b (ppm)	3888.4	4359.6	4940.2	5667.6	6595.5	7799.6
\bar{X} (ppm)	3683.5	4109.9	4630.9	5277.7	6094.5	7144.2
p_f (kPa)	2200	2176	2152	2128	2104	2080
p_b (kPa)	2176	2152	2128	2104	2080	2056
$\Delta\pi_m$ (kPa)	264.3	294.5	331.2	376.7	433.9	506.9
Δp_m (kPa)	2087	2063	2039	2015	1991	1967
RR	0.1043	0.1130	0.1230	0.1346	0.1478	0.1626
SR	0.9527	0.9511	0.9492	0.9469	0.9439	0.9399

in the first and second stages, are shown in Tables 2a and b.

The permeability model is used to predict the system profiles. Design parameters used in the calculations are shown in Table 3. As is shown, the module membrane area is set equal to 0.01 m². The values for the salt and water permeability

are adjusted to obtain the same overall system recovery, product salinity, and product production rate. The system profiles as predicted by the permeability model are shown in Tables 4a and b. The model results agree well with field data, where the predicted product flow rate and salinity are equal to 234.1 m³/day and 314 ppm, respectively.

Table 4b
Predicted profiles by the permeability model for the second stage in Sharjah plant

Module number	7	8	9	10	11	12
Q_f (m ³ /s)	1.65E-04	1.56E-04	1.46E-04	1.37E-04	1.29E-04	1.21E-04
Q_b (m ³ /s)	1.56E-04	1.46E-04	1.37E-04	1.29E-04	1.21E-04	1.14E-04
Q_p (m ³ /s)	9.63E-06	9.24E-06	8.84E-06	8.41E-06	7.98E-06	7.53E-06
X_f (ppm)	7799.6	8253.4	8742.1	9267.2	9830.1	10,431.4
X_p (ppm)	470.3	517.7	572.1	635.0	707.8	792.7
X_b (ppm)	8253.4	8742.1	9267.2	9830.1	10,431.4	11,070.7
\bar{X} (ppm)	8019.7	8490.3	8996.5	9539.8	10,121.2	10,740.8
p_f (kPa)	2056	2032	2008	1984	1960	1936
p_b (kPa)	2032	2008	1984	1960	1936	1912
$\Delta\pi_m$ (kPa)	567.1	598.9	632.8	668.9	707.1	747.3
Δp_m (kPa)	1943	1919	1895	1871	1847	1823
RR	0.0583	0.0594	0.0604	0.0612	0.0618	0.0622
SR	0.9397	0.9373	0.9346	0.9315	0.9280	0.9240

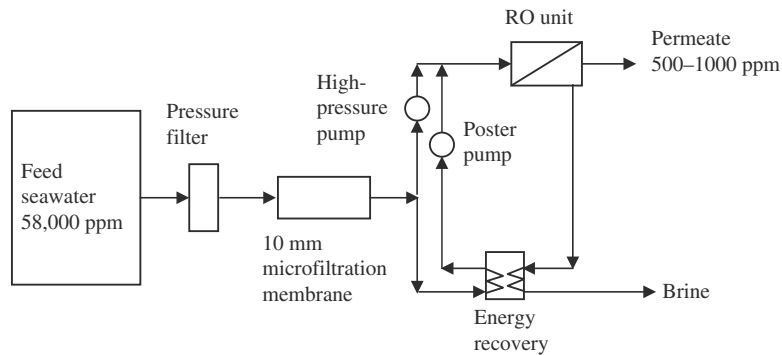


Fig. 2. Schematic of the Qatar SWRO plant.

3.4. Performance analysis of the Qatar SWRO plant

The second illustration is made for a pilot scale RO plant, which is built to test the feasibility of handling high-salinity feed seawater (58,000 ppm), see Fig. 2. The plant has a daily production capacity of 223.2 m³/day. The permeate salinity varies over a range of 500–1000 ppm. The system contains two sets of five pressure vessels and each contains three RO modules. The two sets are placed in series.

Table 5 shows comparison of the semi-empirical model predictions against the collected field data for the RO unit. As is shown, there is an excellent agreement between the field data and the model predictions. The calculated system profiles, which include flow rates, salinity, pressure, and osmotic pressure across the modules, are shown in Table 6.

The design parameters used to solve the permeability model are shown in Table 7. The calculations are performed over six modules in series. The input parameters, which include the membrane module area, the salt permeability, and the water permeability are adjusted in order to obtain an average salt reject of 0.99 and an average permeate recovery per module of 6.2%. The model predictions give an average permeate salinity of 417 ppm, which is close to the lower

limit set by the manufacturer. The flow rate, concentration, pressure, permeate recovery, and salt rejection profiles across the membrane modules are shown in Table 8.

Table 5
Comparison of actual field data and semi-empirical model predictions for the Qatar SWRO plant

Parameter	Actual	Model
Total permeate flow rate (m ³ /day)	223.5	223.5
Feed salinity (ppm)	58,000	58,000
Permeate flow rate per module (m ³ /day)	8.91	8.91
Salt rejection ratio per module	0.983–0.99	0.99
Feed pressure (kPa)	7500	7500
Pressure drop per module (kPa)	NA	24
Permeate pressure (kPa)	101	101
System temperature (°C)	25	25
Average molecular weight of salt	NA	33
<i>Design results</i>		
Number of vessels	5	5
Total permeate flow rate per pressure vessel (m ³ /day)	44.64	44.64
Number of modules	30	30
Average permeate salinity (ppm)	500–1000	693.6
Total system recovery (%)	36.5	36.5

Table 6
Profiles of Qatar SWRO plant predicted by the semi-empirical model

Module number	1	2	3	4	5	6
Q_f (m ³ /day)	122.40	113.49	105.22	97.56	90.45	83.87
Q_b (m ³ /day)	113.49	105.22	97.56	90.45	83.87	77.76
Q_p (m ³ /day)	8.91	8.26	7.66	7.10	6.59	6.11
X_p (ppm)	580	625.0995	673.7058	726.0916	782.5509	843.40023
X_f (ppm)	58,000.00	62,509.95	67,370.58	72,609.16	78,255.09	84,340.02
X_b (ppm)	62,509.95	67,370.58	72,609.16	78,255.09	84,340.02	90,898.11
\bar{X} (ppm)	60,169.76	64,848.43	69,890.89	75,325.45	81,182.59	87,495.16
p_f (kPa)	7500	7476	7452	7428	7404	7380
p_b (kPa)	7476	7452	7428	7404	7380	7356
$\Delta\pi_m$ (kPa)	4476.31	4824.38	5199.51	5603.81	6039.55	6509.18
Δp_m (kPa)	2910.69	2538.62	2139.49	1711.19	1251.45	757.82

Table 7
Simulation parameters for the permeability model for the Qatar SWRO plant

Simulation parameter	Value
Membrane area (m ²)	0.05
Salt permeability (kg/s ppm m ²)	1×10^{-8}
Water permeability (m ³ /s kPa m ²)	7×10^{-7}
Permeate pressure (kPa)	101
System temperature (°C)	25
Average molecular weight of salt (kg/kmole)	33
Pressure dropper module (kPa)	24

Table 8
Profiles of the Qatar SWRO plant predicted by the permeability model

Module number	1	2	3	4	5	6
Q_f (m ³ /s)	1.42E-03	1.30E-03	1.20E-03	1.12E-03	1.05E-03	9.99E-04
Q_b (m ³ /s)	1.30E-03	1.20E-03	1.12E-03	1.05E-03	9.99E-04	9.58E-04
Q_p (m ³ /s)	1.15E-04	9.84E-05	8.25E-05	6.73E-05	5.35E-05	4.16E-05
X_f (ppm)	58,000	63,101.2	68,225.7	73,216.3	77,860.0	82,060.8
X_p (ppm)	230.9	292.4	376.6	490.7	655.2	882.0
X_b (ppm)	63,101.2	68,225.7	73,216.3	77,860.0	82,060.8	85,596.3
\bar{X} (ppm)	60,442.7	65,563.0	70,632.5	75,466.3	79,904.8	83,790.9
p_f (kPa)	7500	7476	7452	7428	7404	7380
p_b (kPa)	7476	7452	7428	7404	7380	7356
$\Delta\pi_m$ (kPa)	4523.0	4903.0	5277.5	5632.1	5953.1	6228.0
Δp_m (kPa)	7387	7363	7339	7315	7291	7267
RR	0.0809	0.0756	0.0685	0.0600	0.0508	0.0416
SR	0.9960	0.9954	0.9945	0.9933	0.9916	0.9893

4. Conclusions

Analysis of the design and operating characteristics is performed for two types of RO plants. The first plant is located in Sharjah and operates on low-salinity brackish water. The second plant is located in Qatar and is designed to desalinate high-salinity seawater for irrigation purposes. The analysis is performed using two models. The first is semi-empirical, which requires knowledge of the membrane recovery ratio and the membrane salt rejection. The second is the permeability model, which requires definition of the water and salt permeability coefficients. Predictions of

the two models are in good agreement with the collected field data for the two plants. Agreement is found for predictions of the total production rate, product salinity, overall recovery ratio, overall salt rejection, and the required number of membrane modules and pressure vessels.

The semi-empirical model is rather simple and does not require iterative solution. It provides flow rate, concentration, and pressure profiles. However, its predictions do not include an important design parameter, which is the module membrane area. This important design parameter is hidden in the module recovery ratio and salt rejection ratios. On the other hand, the permeability model includes explicitly the module permeation area, which appears in the flux equations for water and salt. This knowledge requires simultaneous definition of the permeability coefficients for water and salt. Both models provide useful tools to analyze plant data and to understand variations in system performance.

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Nomenclature

A	Membrane area, m ²
MW	Salt molecular weight, kg/kmole
p_b	Brine pressure, kPa
p_f	Feed pressure, kPa
p_p	Permeate pressure, kPa
P_w	Water permeability, m ³ /(m ² s kPa)
P_s	Salt permeability, kg/(m ² s ppm)
Δp_m	Pressure drop across the membrane, kPa
$\Delta\pi_m$	Osmotic pressure drop across the membrane, kPa
Q_b	Brine flow rate, m ³ /day
Q_f	Feed flow rate, m ³ /day

Q_p	Total production capacity, m ³ /day
R	Universal gas constant, $R = 8.314$ kPa m ³ /kmole K
RR	Permeate recovery, dimensionless, $RR = Q_p/Q_f$
SR	Salt rejection, dimensionless, $SR = (1 - X_p/X_f) \times 100$
T	Temperature (K)
X_f	Feed salinity entering membrane module, ppm
X_b	Salinity of brine stream leaving membrane module, ppm
X_p	Salinity of permeate stream leaving membrane module, ppm
\bar{X}	Average salinity in the feed compartment, ppm

Greek letters

π	Osmotic pressure, kPa
ρ	Water density, kg/m ³

Subscripts

b	Brine
f	Feed
m	Membrane
p	Permeate

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